Limitations on Stochastic Localization Models of State Vector Reduction

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Recently, Ghirardi, Rimini, Weber, and Pearle have proposed a stochastic modification of the Schrödinger equation which dynamically suppresses coherent superpositions of macroscopically distinguishable states and so avoids the infamous cat paradox. We show that the modified dynamics reduces the state vector completely only in the limit of infinite time, and therefore, for any finite time t, no objective local property can be meaningfully assigned to measurement outcomes. Since a physical mechanism giving rise to stochastic spontaneous localizations of the state vector is lacking, we argue that the model (however heuristically interesting) turns out to be *ad hoc*. Finally, we discuss consequences of this latter feature in association with the elusiveness of the two new 'constants of nature' appearing in the model.

1. INTRODUCTION

The relation between classical and quantum mechanics constitutes one of the foundational problems in the interpretation of quantum theory. This problem is particularly well exemplified in the quantum measurement process, where an atomic system (S) interacts with a macroscopic apparatus (A). In such a case, the Schrödinger equation gives rise to linear superpositions of macroscopically distinguishable states for the correlated system $S + A$. The embarrassing implication seems to be that a complete derivation of the definite and separable (local) character of macroscopic objects from the principles of quantum mechanics becomes impossible.

The 'orthodox' response to the problem was to introduce into the theory an additional assumption, the projection postulate, according to which the coherent superposition state is reduced "into an eigenstate of the dynamical variable that is being measured" (Dirac, 1947, p. 36). There has

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been developed, however, during the years a widespread agreement about the fact that the orthodox solution is inadequate. In particular, it has been realized that the projection postulate requires the acceptance of a splitting between measured system and measuring apparatus (or between quantum and classical) which turns out to be 'suspiciously shifty' (e.g., Bell, 1990). Furthermore, the projection postulate is incompatible with the Schrödinger equation of motion and its importation into the axiomatic structure of quantum theory induces a peculiar duality with respect to the time evolution; while Schrödinger's dynamics is linear and deterministic, the projection postulate expresses a nonlinear and stochastic behavior for state vector reduction. One may conceive, then, that a natural reconciliation of these two mutually incompatible types of evolution may be obtained by considering the possibility of nonlinear and stochastic modifications of the standard Hamiltonian dynamics.

2. STOCHASTIC DYNAMICAL STATE VECTOR REDUCTION

Recently, Ghirardi, Rimini, Weber, and Pearle (GRWP) have proposed such a modification of the usual theory in which state vector reduction is described as an objective physical process. The term 'objective' refers to the authors' refusal to attribute a peculiar role to the observer or to accept the 'shifty split' between quantum and classical behavior in micro–macro interactions. Instead, the GRWP framework aims at constructing a single fundamental dynamics governing all phenomena by unifying the two contradictory types of evolution we discussed above into a single propagation law. Such a monistic description offers also the possibility of ascribing individual reality to the wave function as corresponding to the state of a single system in nature.

As one might expect, the GRWP unified dynamics restricts the absolute validity of Schrödinger's equation. The latter is regarded as the limiting case of a more general equation of motion which leaves the evolution of systems in the quantum domain practically unaltered, but which aims at suppressing the embarrassing superpositions of differently located states at the macroscopic level. The elimination of coherence is supposed to occur dynamically at the individual level of description according to an intrinsically stochastic and irreversible dynamics which induces spatial spontaneous localizations on the wave function.

The idea of spontaneous localization has been formulated in two main approaches. The earliest and most intuitive approach (DSL) introduces finite stochastic changes for the state vector in the form of discontinuous jumps occurring around appropriate positions (Ghirardi *et al.*, 1986). For this reason, this kind of stochasticity has also been called a 'hitting'

process. In the more refined continuous approach (CSL) the sudden jumps are replaced by a continuous stochastic evolution—a sort of Brownian motion for the state vector—in Hilbert space (Pearle, 1989; Ghirardi *et al.*, 1990). The CSL model is more general and powerful. However, we first consider the DSL type of dynamics because of its more immediate physical content.

2.1. Discontinuous Spontaneous Localization

The basic assumption underlying the DSL model is that any particle, whether isolated or part of a physical system, is subjected at randomly distributed times to approximate spatial localizations (hittings) which act against the expansion of the system's wave function due to the Schrödinger evolution. As a consequence of a localization around a point x in physical space (we consider a one-dimensional case for simplicity), an initial wave function $|\Psi\rangle$ representing a particle in position basis collapses instantaneously into a new wave function $|\Psi_x\rangle$ according to,

$$|\Psi\rangle \rightarrow |\Psi_x\rangle = |\Phi_x\rangle/\|\Phi_x\|$$

$$|\Phi_x\rangle = \left(\frac{a}{\pi}\right)^{1/4} \exp\left[-\frac{1}{2}a(q-x)^2\right]|\Psi\rangle$$

(1)

where q is the position operator of the particle undergoing the localization process and a is a parameter of length dimensions whose meaning is that $1/\sqrt{a}$ represents the accuracy (or width) of the localization around the 'hitting center' x. The characteristic localization length $1/\sqrt{a}$ is assumed to take the value 10^{-5} cm.

Equation (1) shows that the GRW discontinuous approach consists in multiplying the initial wave function by a Gaussian (jump) function which is distributed around x with spread $1/\sqrt{a}$. In turn this multiplication corresponds to the effect of an approximate localization: if the particle is initially already located within an interval $1/\sqrt{a}$ around x, its wave function is well localized and therefore remains practically unaltered by the hitting process; if, on the contrary, the initial wave function spreads over a distance larger than the characteristic localization length, it is compressed by the process.

The probability density for such a hitting to occur at any particular point x is assumed to be

$$P(x) = \|\Phi_x\|^2$$

(2)

Thus localization processes are most likely to appear at those places where the initial wave function is largest. Note that the GRW probability prescription is analogous to Born's probability rule for the outcomes of a

measurement, in that relation (2) ensures that hittings occur with higher probability at those regions where in standard quantum mechanics there is a higher probability of finding the particle.

Finally, it is also assumed that hittings occur at random times, according to a Poisson distribution, with mean frequency $\lambda = 10^{-16} \sec^{-1}$ and that in the time interval between two successive hittings the state vector evolves following the Schrödinger equation.

In the DSL approach just described, how the reduction mechanism works is quite transparent. As explained, it is incorporated in the basic principle of the DSL dynamics. The repeated application of process (1) with the probability rule (2) strives to suppress coherent superpositions of macroscopically distinguishable states separated by a distance much larger than $1/\sqrt{a}$ to one of its components. The probability that a particular component survives is proportional to its weight in the initial superposition. This guarantees that the probabilistic predictions of the orthodox measurement theory are reproduced. More precisely, as we shall see, they are reproduced up to small but irreducible anomalies akin to the anomalous spontaneous localizations.

2.2. Continuous Spontaneous Localization

The CSL model, based on the consideration of continuous diffusion processes in Hilbert space, avoids the physically unsatisfactory instantaneous changes of the state vector, formulates the stochastic part of the evolution principle through a single unified equation, and preserves the symmetry properties of the state vector in the case of systems with identical particles. In this formulation, the usual Hamiltonian dynamics is subjected to a non-Hermitian, randomly fluctuating potential which depends upon a set of white noise functions $\{w(x, t)\}$, that is to say, tiny Gaussian random processes, coupled to a set of self-adjoint commuting operators $\{A(x)\}$ whose joint eigenvectors (as explained shortly below) represent the density of particles around x.

The state vector obeys a linear stochastic differential equation of the Itô type

$$d|\Psi_w(t)\rangle = -iH|\Psi_w(t)\rangle + \left[\int dx\, A(x)w(x, t) - \gamma \int dx\, A^2(x)\right]|\Psi_w(t)\rangle \quad (3)$$

with the random functions $w(x, t)$ satisfying the following expectation values:

$$\langle w(x, t)\rangle = 0, \qquad \langle w(x, t)w(x', t')\rangle = \gamma\delta(x - x')\delta(t - t') \quad (4)$$

Thus, while $w(x, t)$ is permitted to fluctuate positively as well as negatively with equal likelihood, its fluctuations tend to be statistically correlated

through the parameter γ which controls the strength of the stochastic process. It is interesting to note that under an appropriate choice of γ and in the infinite-frequency limit ($\lambda \to \infty$, $a \to 0$) the discontinuous process (1) can be shown to transform itself into the continuous process (3) (Nicrosini and Rimini, 1990). From this point of view equation (3) acquires an immediate physical content as describing a succession of 'tiny spontaneous localizations' which continuously strive to reduce the state vector into one of the common eigenstates of the set of commuting operators $\{A(x)\}$.

In effect, the operators $A(x)$ determine, for different values of the parameter x, the eigenmanifolds on which reduction takes place. Their specific form, to so-called 'preferred basis,'[2] is assumed to be defined by

$$A(x) = \left(\frac{a}{2\pi}\right)^{3/2} \int dq \, n^{\dagger}(q)n(q)e^{-a(q-x)^2/2} \tag{5}$$

where $n^{\dagger}(q)$ and $n(q)$ are the creation and annihilation operators for a particle at point q in physical space. Then the eigenvalues of $A(x)$ can be taken as representing the average number of particles contained within a sphere centered at x with volume of the order of $a^{-3/2}$ and radius $a^{-1/2}$. Particle number (or macroscopic densities) is thus the proposed carrier for reduction in CSL. The consequence is that a state vector in a superposition of states describing a macroscopic collection of particles in different places separated by a distance $\gg 1/\sqrt{a}$ will reduce to a state with a definite number of particles. The particular eigenstate $|\Psi_w(t)\rangle = |\alpha_i\rangle$ into which the CSL dynamics drives the state vector is determined by a specific realization $w_i(t)$ of the Brownian process.

Provided that different sample functions $w_i(t)$ generate different norms for $|\Psi_w(t)\rangle$, GRWP, in order to get a consistent theory, resort to an assumption parallel to the one made within DSL for the probability density of the hitting positions. They assume that the probability for a specific realization of the stochastic process w_i, or equivalently of the state vector $|\Psi_w(t)\rangle$, is not the 'raw' one $P_{raw}[w_i]$ associated with the white noise distribution of equation (4), but is given by the 'cooked' probability $P_C[w_i]$

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[2]Because state vector reduction in the GRWP model is regarded as a physical process that is actually taking place, the choice of the preferred reduction basis is restricted by physical considerations. For example, if one chose momentum as the reduction eigenmanifolds (Benatti *et al.*, 1988), then the internal motion of semirigid bodies would be massively disrupted by the reduction process, due to the ensuing wide position spread. This and other considerations of a mathematically inspired nature point toward the GRWP choice for a position reduction basis, as exhibited in (5). Clearly, the selection of a preferred basis affords a certain economy to the model; it makes possible the elimination of the two types of physical evolution. The price to pay, however, is that Lorentz invariance does not rest comfortably with the idea that position enjoys a special role in the space-time manifold (Karakostas and Dickson, 1995).

[as Pearle (1989) explains], which depends nonlinearly on the state vector at time t according to

$$P_{\mathrm{C}}[w_i] = P_{\mathrm{raw}}[w_i] \left\| \left| \Psi_w(t) \right\rangle \right\|^2 \tag{6}$$

The probability rule (6) makes, for a given $P_{\mathrm{raw}}[w_i]$, state vectors with large norm to weigh more, i.e., more likely to occur, in measurement situations. In other words, according to this prescription, the most probable realizations of the stochastic process $w(x, t)$ are those which guide the evolution of $|\Psi\rangle$ in such a way that the state vector becomes localized around those positions x where $|\Psi_x|$ is large, while avoiding regions where $|\Psi_x|$ is small.

3. STOCHASTIC LOCALIZATION AND INDEFINITENESS OF MEASUREMENT OUTCOMES

To explain how the CSL stochastic evolution continuously strives to reduce the state vector into one of the common eigenmanifolds of the operators $\{A(x)\}$, we consider a simple quantitative example in which only one operator A appears in equation (3), i.e.,[3]

$$d\left| \Psi_w(t) \right\rangle = -iH\left| \Psi_w(t) \right\rangle + [Aw(t) - \gamma A^2]\left| \Psi_w(t) \right\rangle \tag{7}$$

If we disregard the Hamiltonian part, since we want to study the pure reduction properties of the CSL dynamics, the solution of equation (7) is

$$\left| \Psi_{\mathrm{B}}(t) \right\rangle = \exp[AB(t) - \gamma A^2 t]\left| \Psi(0) \right\rangle \tag{8}$$

where $B(t)$ is a Brownian motion function associated to the white noise potential through

$$B(t) = \int_0^t w(t)\, dt \tag{9}$$

with 'raw' probability density

$$P_{\mathrm{raw}}[B(t)] = \frac{1}{N} e^{-B^2/2\gamma t} \tag{10}$$

N being a normalization factor.

The change in the notation for the state vector from $|\Psi_w(t)\rangle$ used in equation (7) to $|\Psi_{\mathrm{B}}(t)\rangle$ underlines the fact that in the Brownian case the state at time t does not depend on the specific sample function w_i in the interval $(0, t)$, but only on its integral $B(t)$ of equation (9).

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[3]For simplicity we deprive A of its physical meaning and treat it as a purely mathematical object in discrete space.

Let us further assume that the operator A is spanned by only two eigenmanifolds M_l, M_r with eigenvalues l, r corresponding to macroscopically different outcomes, so that the initial state vector of the system under measurement has the form

$$|\Psi(0)\rangle = L|\Psi_l\rangle + R|\Psi_r\rangle \tag{11}$$

Then, the time evolution of this two wave-packet system can be written, according to equation (8), as

$$|\Psi_B(t)\rangle = L \exp[lB(t) - \gamma l^2 t]|\Psi_l\rangle + R \exp[rB(t) - \gamma r^2 t]|\Psi_r\rangle \tag{12}$$

where the sum of the two state vectors at time 0 becomes, at time t, a state vector which is the sum of the two evolved state vectors.

The solution of equation (12) for $|\Psi_B(t)\rangle$ depends upon a particular realization of the Brownian motion $B(t)$ that drives the state vector. In the absence, however, of any knowledge of the actual $B(t)$, we resort to the cooking prescription (6), which assigns the following probability distribution to the Brownian process at time t:

$$P_C[B(t)] = P_{\text{raw}}[B(t)] \left\| |\Psi_B(t)\rangle \right\|^2$$

$$= \frac{1}{N} \left\{ |L|^2 \exp - \frac{[B(t) - 2l\gamma t]^2}{2\gamma t} + |R|^2 \exp - \frac{[B(t) - 2r\gamma t]^2}{2\gamma t} \right\} \tag{13}$$

In the derivation of P_C we have used the approximation that the squared norms of the two packets are uncorrelated, i.e., that they behave precisely the same way as the squared amplitudes at two distant points.

According to equation (13), the most probable values of $B(t)$ occur when either or both exponents in (13) are small, i.e., when the following conditions are satisfied:

$$|B(t) - 2l\gamma t| < (2\gamma t)^{1/2}, \qquad |B(t) - 2r\gamma t| < (2\gamma t)^{1/2} \tag{14}$$

It is evident from relations (14) that the Brownian process $B(t)$ fluctuates with overwhelming probability around an interval of width $(2\gamma t)^{1/2}$ centered on either the value $2l\gamma t$ or the value $2r\gamma t$.

When, for instance, $B(t)$ takes a value near $2r\gamma t$, the norm of the $|\Psi_r\rangle$ packet grows exponentially with rate $\exp(r^2 \gamma t)$

$$\langle \Psi_r | \Psi_B(t) \rangle \simeq e^{r^2 \gamma t} R \tag{15}$$

whereas the norm of $|\Psi_l\rangle$ decreases,

$$\langle \Psi_l | \Psi_B(t) \rangle \simeq e^{-\gamma t(r-l)^2} e^{r^2 \gamma t} L \tag{16}$$

with a rate depending on the squared difference of the eigenvalues. Consequently, the associated (normalized) state vector $|\Psi_B(t)\rangle$ of equation (12), subject to the particular Brownian function of $B(t) \simeq 2r\gamma t$, is driven into the eigenmanifold M_r corresponding to the eigenvalue r of A with probability approaching $|R|^2$ as the 'tails' in the initial superposition of the two packets are exponentially reduced according to $\exp[-\gamma t(r-l)^2]$. In fact, it is easy to see from equations (15) and (16) that the ratio of the unnormalized squared norms of the two packets is

$$\frac{|\langle\Psi_l|\Psi_B(t)\rangle|^2}{|\langle\Psi_r|\Psi_B(t)\rangle|^2} \simeq e^{-2\gamma t(r-l)^2}\frac{|L|^2}{|R|^2} \tag{17}$$

A similar relation holds in the case when the stochastic variable $B(t)$ takes a value near $2l\gamma t$.

This picture makes apparent that the determination of a specific measurement–outcome in the GRWP theory is achieved through a particular realization of the Brownian process. In the above example, for instance, $B(t)$ generates a sort of Brownian motion for the state vector, so that $|\Psi_B(t)\rangle$ undergoes a diffusion process with two possible 'drifts' corresponding to two incompatible macroscopic situations ('right' and 'left' pointer readings). It is the choice of the drift, randomly made with probability weights (almost) reproducing the quantum mechanical statistics, that drives the state vector into a particular 'pointer reading' eigenstate, and the further stochasticity plays no significant role.

However, the GRWP dynamical reduction theory presents an element of inconsistency with respect to a standard criterion of adequacy imposed on any theory of measurement, namely, that *theoretical measurement times must approximate as closely as possible to actual measurement times*. The embarrassing result obtained by the CSL model is that it requires the measurement to be *arbitrarily long* in duration. It can be immediately seen from equation (17) that its right-hand side tends to zero, and therefore the norm of the packet $|\Psi_r\rangle$ becomes dominant, when t is sufficiently large ($t \gg \gamma^{-1}$). In the two-wave-packet system we examined above, the initially superposed state vector (11) can be driven by the CSL reduction dynamics into a 'fixed pointer position,' that is to say, either into the manifold M_l or M_r, only in the unattainable mathematical limit $t \to \infty$.

For any finite time t, no one of the packets evolved from this initial superposition can be identified with an exact eigenstate of the pointer's position observable. This is a direct consequence of the model's stochastic (white noise) assumptions (multiplication of the state vector by tiny Gaussian random processes), according to which for all values of $B(t)$ which have an overwhelming probability to occur [cf. equation (14)], the norm of the corresponding dominant (normalized) state vector never

grows to unity for all finite times. For instance, when $B(t)$ fluctuates around the value $2r\gamma t$

$$|\langle \Psi_r | \Psi_B(t) \rangle_N|^2 = |R|^2/[|R|^2 + |L|^2 \, e^{-2\gamma t(r-l)^2}] \tag{18}$$

the squared amplitude approaches the value 1 exponentially with time, so the associated state vector will always have a nonzero amplitude interfering with that of the $|\Psi_l\rangle$ component which decays to 0 for $t \to \infty$. Hence each state vector in the ensemble remains always in a superposition whose existence prevents one from asserting (contrary to macroscopic experience) that the pointer occupies a definite position in space.

The indefiniteness of the pointer's position is also suggested on generic physical grounds by the fact that the GRWP model, due to the appearance of the non-Hamiltonian terms in the modified evolution equation (3), implies a *violation* of energy conservation. The repeated application of the localization processes will squeeze the Schrödinger spatial development of a wave packet and thus increase the system's energy. The result is a continuous gain in energy by macroscopic bodies, which, although it is so small as to be practically undetectable (Ghirardi and Rimini, 1990), is nonetheless an undesirable feature in a fundamental theory—the connection between symmetry relations of space and time, invariance principles, and conservation laws breaks down—and with respect to the measurement, it prevents in principle the establishment of a final (pointer-reading) equilibrium state.

Both the persistence of the superposed 'tails' and the incompatibility of the spontaneous reduction mechanism with the attainment of equilibrium suggest that within the GRWP picture the objectification process of a measurement, i.e., the emergence of a definite outcome, never takes place ($t = \infty$).[4]

The implication is that if one wishes to attribute objective properties to individual systems in finite time intervals (normally specified by the reaction time of the apparatus at hand), one has to accept, as GRWP do, that the 'outcome r' has been objectively realized even when $|\langle \Psi_r | \Psi_B(t) \rangle|^2$ is not exactly equal to 1 but significantly close to it.

We take the view, however, that a satisfactory way for restoring the concept of independent reality within the dynamical reduction program would be, instead of adopting an imprecise correspondence between theoretical elements and results of observations, to consider the possibility of a more realistic (than white noise) stochastic source which may lead to

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[4]Earlier nonlinear state vector reduction models did not suffer this consequence, by achieving finite reduction times (Pearle, 1976), but did suffer the prediction of superluminal communication.

complete state vector reduction in characteristic measurement times. The realization of the former possibility, however mathematically difficult, is not impossible and Pearle (1993) sketches an approach by means of which this aim might be achieved. No definite conclusion has been reached, however, and further work seems to be necessary if the GRWP model is to be physically generalized.

4. COLLAPSE PARAMETERS AND THEIR CONSEQUENCES

Admittedly, the model of GRWP proposes a new fundamental dynamics for the Schrödinger evolution, which aims at transforming "quantum mechanics into a stochastic mechanics in phase space exhibiting . . . classical features" (Ghirardi *et al.*, 1986, p. 485). Generally speaking, attempts for the modification of standard quantum mechanics face the task of convincing skeptics that the Schrödinger equation, despite its deep mathematical beauty and unprecedented degree of confirmation, is not absolutely valid. In the GRWP case, this problem takes the particular form of providing a physical motivation for the introduction of stochastic spontaneous localizations at a fundamental level of description for what may otherwise appear to be an *ad hoc* modification of the basic evolution of the wave function.

The GRWP model plainly accepts the ordinary interpretation of the wave function as providing the most accurate description of a quantum system. No extra variables are added therefore to the state vector, which remains the only state variable. Extra terms are added, however, to the usual Hamiltonian dynamics. In the modified Schrödinger evolution the state vector interacts with new physical variables described by white noise stochastic processes. As mentioned, it is these diffusion processes which guide probabilistically the state vector into a particular 'pointer-reading' eigenstate. The implication is that probabilistic behavior in the GRWP picture is not exclusively associated with the act of measurement. It is incorporated in the evolution principle of the theory in the form of random spontaneous localizations (jumps). However, no explanation is given as to how randomness emerges at the quantum level of description and consequently the wave function is assigned a role in the stochastic behavior of something still to be determined. The stochastic terms in the evolution equation—held responsible for the occurrence of the spontaneous localizations of the wave function—must be identified with the operation of some physical mechanism [instead of bringing them out of a hat just like that (Bell, 1989)] if the dynamical reduction program is to stand in its own right. Under the present regime, it seems that the GRWP stochastic dynamics requires recourse to some external physical

input in order to account for the definiteness of the world at the macro-scopic level. Such an input might be provided by connecting the reduction mechanism to quantum gravity, as has been suggested in the literature (cf. Diosi, 1992), although novel problems in relation to energy nonconserva-tion may arise.[5]

Appeal to such a physical or otherwise mathematical input is inti-mately connected with the empirical adequacy of the new theory. The GRWP modified dynamics captures the empirical content of standard quantum mechanics in all cases in which the latter has so far been confirmed experimentally, through a careful choice of the numerical values of the two new parameters a and λ [or $\gamma \equiv \lambda(a/4\pi)^{-3/2}$] appearing in the model. Recall that the parameter $\lambda = 10^{-16}\,\mathrm{sec}^{-1}$ refers to the mean frequency of the localization process of a single particle, the parameter a is related to the accuracy of the process, with the value $1/\sqrt{a} = 10^{-5}\,\mathrm{cm}$ determining a new fundamental length making specific the quantum action at a distance (transition from quantum to classical behavior), whereas the other parameter $\gamma = 10^{-30}\,\mathrm{cm}^3\,\mathrm{sec}^{-1}$ in the continuous version expresses the strength of the statistical correlations among the white noise functions in the evolution equation.

It is understood, however, that the values of these quantities—quanti-ties which deserve to be called constants of nature if the GRWP dynamics is taken as describing fundamental physical processes—are not to be determined merely by requiring consistency with known data (essentially by minimizing unpleasant physical implications concerning the mean en-ergy increase or the dissociation probability of atoms), but instead should be determined experimentally with a high degree of accuracy. For example, Zeilinger (1986, p. 23), in testing possible deviations of the state vector evolution from the unitarity of the Schrödinger equation, has obtained as a *lower* limit for the spontaneous reduction time in the case of a two-slit neutron interference pattern, $\tau > 4\,\mathrm{sec}$, provided that the maximum possi-ble interference contrast is experimentally used. Since the GRWP model suggests a characteristic reduction time of $\tau \simeq 10^{16}\,\mathrm{sec}$ for a microscopic system, an improvement by a factor of 10^{15} (!) is needed in the precision of this experiment to become sensitive to the numerical choice made for the

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[5]Another possibility, perhaps also reducible to gravity, is the consideration of the often neglected system–environment interaction. On this basis, Joos and Zeh (1985) derived for the statistical operator of evolution a Markovian 'master equation' of the Itô type leading to destruction of macroscopic coherence. The density-matrix formalism does not, however, allow one to derive classical states at the level of particle trajectories without involving additional assumptions. Nevertheless, Primas (1990) has shown, through the consideration of nonlinear feedback effects, how the environment-induced reduction can be extended to the state vector level of description also.

constants a and λ. This highlights the fact that the prospects of neutron interference experiments that would actually test the GRWP theory being performed in the future are negligible.[6]

Furthermore, it is worthy to note that at the methodological-theoretical level, constants entering physical laws—laws which are applicable in principle to any physical phenomenon—cannot be taken merely as fixed quantities with the sole purpose of facilitating calculations or achieving reductions of wave packets, as in the GRWP case. On the contrary, their epistemological status plays a conceptual role bearing witness to the development of physical science in general.[7] A common example is Planck's constant $\hbar$ of the quantized action, which not only serves as a 'concept synthesizer' (for instance, the relation $E = \hbar\omega$ leads to the idea of wave–particle duality in standard quantum mechanics and further on to Bohr's philosophy of complementarity), but also, through the interelations among the various physical concepts within the theoretical framework (e.g., $p = \hbar k$, $L = \hbar m$, etc.), it can be used as a conceptual tool for building up the new theory. However, no such link of conceptual arrays can be found in the model of GRWP. The two new fixed quantities operate simply as numerical parameters whose suggested values were chosen with an eye to avoiding a contradiction for all intents and purposes with the quantum mechanical predictions for microscopic objects.

Naturally, one would expect that a systematic and physically motivated development of stochastic quantum dynamics should be related to standard Hamiltonian dynamics through some limiting principle which would resemble the 'correspondence principle' relating quantum to classical mechanics. We emphasize in this respect that the limit $\gamma \to 0$ in which the modified dynamics goes into the usual Schrödinger evolution[8] would not do. First, this limiting process is completely circular, bringing one back to the point of departure, that is, the standard quantum dynamics; one posits stochasticity in the wave function to achieve state vector reduction and then arbitrarily removes it to recover the 'Hamiltonian limit.' The vanish-

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[6]Considerations about the experimental detection of GRWP-type decorrelating effects on a SQUID operating in the macroscopic domain result in the same conclusion (Rae, 1990; Gallis and Fleming, 1990).

[7]The whole structure of physics can be parametrized by the three fundamental constants G, c, and $\hbar$, referring, respectively to the three fundamental theories: Newtonian gravity, special relativity, and quantum mechanics; whereas pair combinations of them may be taken to correspond to the partially unified theories of Einstein's general relativity $(G-c)$, Dirac's relativistic quantum mechanics $(c-\hbar)$, and modern quantum cosmology $(G-\hbar)$, or even to a future, fully unified theory for $(G-c-\hbar)$. One may therefore legitimately wonder about the association of the GRWP parametrization $(a-\lambda-\hbar)$ with the rest of physics.

[8]This is immediately apparent at the formal mathematical level if we rewrite equation (3) in the language of statistical operators, i.e., $d\varrho/dt = -i[H, \varrho] - \gamma \int d^3x \, [A(x), [A(x), \varrho]]$.

ing of γ (or λ) is in no way sufficient to guarantee the physical relevance of the limiting theory. If one is to study the logic of the possible limits of a theory, one must start from this theory itself, as expressed within its autonomous system of concepts. In other words, the GRWP theory needs an argument for why the *universal* character of the stochastic evolution fades out for systems commonly governed by Hamiltonian dynamics. Second, the limit of a constant to zero is never realized: neither γ nor $\hbar$ is actually zero. Such a limiting process is, at best, physically obtained when the ratios of certain physical quantities are small (or large) in comparison to the constant which relates the two quantities. For instance, the quantum behavior of a system is considered to be typically approximated by classical particle mechanics when the width of the wave packet is actually large in comparison to the wavelength. It is the relative significance of these two factors with respect to Planck's constant which gives rise to a physically permissible approach to the classical limit. However, the γ (or λ) parameter, as already mentioned, does not enjoy any such interrelational status within the GRWP context. It has been put in by hand from outside the theory to import the right kind of stochasticity into the evolution of the wave function for achieving state vector reduction without appreciably disturbing the quantum mechanical statistics. As things stand, it appears that the deterministic Schrödinger dynamics and the stochastic localization (reduction) dynamics, rather than being related by some guiding physical principle, coexist by decree in the equation of motion.[9]

A further difficulty, associated also with the arbitrariness of the collapse parameters, concerns the assumption about the universal character of the localization process according to which, for a system of N particles, the localization occurs "individually for each constituent of the many-particle system" (Ghirardi, et al., 1986, p. 477). In the frame of quantum mechanics this assumption seems to be dubious at least in two respects if it is to acquire meaning beyond a pragmatic approximate description. For, first, the wave function of an N-body system does not in general define its individual parts, and, second, the question of what is a part of a complex system can only be given a contextual answer. For instance, in a many-particle system, are the parts the molecules, atoms, nucleons, or even quarks or partons? The level at which the spontaneous localization process is supposed to occur is left unspecified and in any case the parameter γ (or λ) should *continually vary in magnitude* according to the kind of particles under consideration if agreement with quantum mechanical predictions is to be maintained.

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[9]Again, a possible connection of the introduced stochasticity to the rest of physics (e.g., gravity, cosmic background radiation, electromagnetism) might pave the way toward a physical generalization of the GRWP theory.

5. FINAL REMARKS

In our view models with spontaneous localization of the state vector are useful to provide an effective phenomenological way of saving the concept of independent reality, that macroscopic objects possess objective local properties not affected by the long-distance coherence of far-away states.

We do not think, however, that they might serve as a fundamental approach to the quantum theory of measurement. As we showed, the magnitude of problems connected with wave packet reduction is more complex than can be expressed by the addition of a stochastic term in the Schrödinger equation. The authors, by doing so, have transferred the apparent duality in the axiomatic structure of quantum mechanics into a single dynamical law which, however, exhibits a dual character still to be explained.

A related, but more general, objection to the GRWP modification of the Schrödinger equation is that it destroys the fundamentality of the present structure of quantum dynamics as it finds expression in universal physical principles like Lorentz invariance and laws of conservation. Clearly, a stochastic modification of the fundamental quantum dynamics will not be ultimately acceptable unless it leads to novel consequences which can be experimentally confirmed.

In the absence of any feasible experimental tests that would actually confirm or falsify the GRWP postulate of stochastic spontaneous localization of the wave function, acceptance of this point of view can only be based on purely conceptual (philosophical) considerations. In this regard, the model of GRWP should be commended for the fact that it does provide an intuitive picture of how quantum events may come about by accommodating our knowledge of microprocesses into a coherent quantum ontology that generally allows a macroobjective description of natural phenomena.

ACKNOWLEDGEMENTS

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